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Econometric modeling for optimal hedging in commodity futures: An empirical study of soybean trading

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ABSTRACT

The optimal hedge ratio (OHR) is basically based on the coefficient of the regression between the change in the spot prices and the change in price of the hedging instrument. The traditional constant hedge ratio based on the ordinary least square (OLS) technique has been avoided by the researchers being an inappropriate; it ignores the heteroscedasticity which often exists in price series. In other words, the hedge ratios will certainly vary over time as the conditional distribution between cash and futures prices changes. It has been recognized that time varying coefficient (TVC) model outperforms the static coefficient (SC). As an illustration, the future and spot price of Soybean have been considered for the contracts maturing in December, 2011; June, 2012; December, 2013; April, 2013. The hedge ratio has been estimated for all the contracts by using OLS method, GARCH-BEKK, GARCH-VECH and Kalman filter methodology.

Keywords: Optimal hedge ratio, Kalman filter, Time varying coefficient models

Hedging in future market has emerged out as an interesting area of research in the recent time mainly because of expansion of derivative markets. In a volatile financial and economic situation, element of risk has become more important in decision making. In hedging process, the expected future value has been locked in and thereby reduces the effect of volatility. The complete elimination of risk has not been the market strategy and thus some amount of risk is allowed to exist. The traditional constant hedge ratio based on the ordinary least square (OLS) technique has been avoided by the researchers being an inappropriate; it ignores the heteroscedasticity which often exists in price series. In other words, the hedge ratios will certainly vary over time as the conditional distribution between cash and futures prices changes. Autoregressive conditional heteroscedastic (ARCH) and the generalized ARCH (GARCH) models have been widely used to estimate time-varying hedge ratios in the futures markets. The optimal hedge ratios estimated by means of the

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GARCH models is time varying, because these models take into consideration the time-varying distribution of the cash and futures price changes. The optimal hedge ratio (OHR) is basically based on the coefficient of the regression between the change in the spot prices and the change in price of the hedging instrument. It has been recognized that time varying coefficient (TVC) model outperforms the static coefficient (SC). Bera *et al*. (1997) dealt with the estimation of optimal hedge ratios. Hatemi *et al*. (2006) considered crucial input in the hedging of risk is the optimal hedge ratio; which has been defined as the relationship between the price of the spot instrument and that of the hedging instrument.

It has been considered that the expected relationship between economic or financial variables may be better captured by a time varying parameter model rather than a fixed coefficient model, the optimal hedge ratio, therefore, can be one that is time varying rather than constant. Choudhry (2007) investigated the hedging effectiveness of time-varying hedge based on four different versions of the GARCH models. The GARCH models applied are the standard bivariate GARCH, the bivariate BEKK GARCH, the bivariate GARCH-X and the bivariate BEKK GARCH-X. The GARCH-X and the BEKK GARCH-X models are uniquely different from the other two models because they takes into consideration effect of the short-run deviations from the long-run relationship between the cash and the futures prices on the second conditional moments of the bivariate distribution of the variable. The hedging effectiveness is estimated and compared by checking the variance of the portfolios created using these hedge ratios.

The lower the variance of the portfolio, the higher is the hedging effectiveness of the hedge ratio. Ibrahim *et al.* (2010) in this study, the time-varying hedge ratio was analysed using the State Space model (Kalman Filter) on daily Kuala Lumpur Composite Index (KLCI) and Kuala Lumpur Future Index (KLFI) from April 2005 to March 2008. Comparison between the static and time-varying hedge ratio and forecast performance is done to analyse the efficiency of the time-varying estimates. Our results show that for forecasting purposes the State Space model has the ability to forecast better when 30 days of forecast horizon are used. The volatility of the time varying hedge ratio is relatively low, but the static estimate of the hedge ratio overestimates the amount of the KLFI futures contract needed to hedge the KLCI. Hatemi *et al*. (2010) showed that the performance of a stochastic hedge ratio is different than the performance of a constant hedge ratio even in the situations in which the mean value of the stochastic hedge ratio is equal to the hedge ratio with a constant structure. This study suggests and demonstrates the use of the Kalman Filter approach for estimating time varying hedge ratio– a procedure that is statistically more efficient and with better forecasting properties. The investigation also includes the application of GARCH models for estimating time varying hedge ratio.

Data and Model specification

Data on future trading in Soybean from November 2012 to April 2013 at NCDEX platform was performed .The spot price pertain to Indore Market for the same period.

This study employs the conventional ordinary least square (OLS),

$$r_{S,j} = \alpha + \beta r_{f,j} + \varepsilon_j \tag{1}$$

Where $r_{S,j}$ is the commodity spot return and $r_{f,j}$ is the commodity futures return. The OLS estimator is

$$\beta^* = \frac{\sigma_{sf}}{\sigma_f^2} \tag{2}$$

Where β^* is the optimal hedge ratio which will maximize the utility function of an investor who faces the mean–variance expected utility function. This conventional hedging strategy assumes that the investor holds on unit the long position in the spot commodity market. To maximize his utility as well as minimize the variance of his long position, he holds the β^* unit of short position in the futures market. When β is one, it is called naïve hedge strategy.

Since the joint distribution of commodity spot and futures market could be time-varying, we also consider the alternative models from the multivariate GARCH family. The simplified diagonal VECH GARCH (1,2) (DVEC GARCH) model, introduced by Bollerslev *et al*. (1988).

$$r_{Sj} = \alpha_S + e_{S,j} \tag{3}$$
$$r_{f,j} = \alpha_f + e_{f,j}$$

$$\begin{bmatrix} e_{s,j} \\ e_{f,j} \end{bmatrix} | \Psi_{t-1} \sim N(0,H) \tag{4}$$

$$H_t = U + A \otimes e_{t-1}e_{t-1} + B \otimes e_{t-2}e_{t-2} + C \otimes H_{t-1} \tag{5}$$

$$\begin{bmatrix} h_{ssj} & 0 \\ h_{fsj} & h_{ffj} \end{bmatrix} = \begin{bmatrix} u_{ss} & 0 \\ u_{fs} & u_{ff} \end{bmatrix} + \begin{bmatrix} a_{ss} & 0 \\ a_{fs} & a_{fj} \end{bmatrix} \begin{vmatrix} e_{sj-1} & e_{sj-1} & 0 \\ e_{fj-1} & e_{sj-1} & e_{fj-1} & e_{fj-1} \end{vmatrix}$$
$$\begin{vmatrix} b_{ss} & 0 \\ b_{fs} & b_{ff} \end{vmatrix} \begin{vmatrix} e_{sj-2}e_{sj-2} & 0 \\ e_{fj-2}e_{sj-2} & e_{fj-2}e_{fj-2} \end{vmatrix} + \begin{vmatrix} c_{ss} & 0 \\ c_{fs} & c_{ff} \end{vmatrix} \begin{vmatrix} h_{ssj-1} & 0 \\ h_{fsj-1} & h_{ffj-1} \end{vmatrix} \tag{6}$$

Where Equation (3) is the mean equation of the model; e_j is the innovation term, which follows a normal distribution with mean zero and conditional variance H_t; Ψ_t information set at time t-1; and $\otimes$ is the Hadamard product. Equation (4) and (5) show that conditional variance follows an ARMA (1,2) process, this depends on its last-period variance and last-period squared residual. As shown in Equation (6), we only consider the lower triangular part of the symmetric metrics of U.A, and B. The covariance matrix must be positive semi-definite (PSD), but H in the DVEC model cannot be guaranteed to be PSD. Therefore, we adopt the fourth model-Matrix Diagonal GARCH (1,2) Model, modified from Bollerslev *et al*. (1994):

$$H_t = UU^{\wedge} + AA^{\wedge} \otimes e_{t-1} e_{t-2}^{\wedge} + \ c \otimes H_{t-1} \tag{7}$$

Where b is a scalar. Equation (7) is a simple PSD version of the DVEC model. Although the Matrix Diagonal model has PSD covariance matrices, the dynamics in the covariance matrices are still restricted. Engle (1995) proposed the famous BEKK (Baba-Engle-Kraft-Kroner) GARCH (1,2) model, which would be our fifth model,

$$H_t = UU^{\wedge} + A(e_{t-1}\, e_{t-1})A^{\wedge} + B(e_{t-2}\, e_{t-2})B^{\wedge} + CH_{t-1}C^{\wedge} \tag{8}$$

Where Equation (8) not only guarantees the PSD but also allows unrestricted matrices where variances of the two variables have concurrent impact on each other by estimating two more parameters a_{sf} and b_{sf}. The constant conditional correlation – CCC GARCH (1,2) model, suggested by Bollerslev (1990), which assumes a time-invariant correlation, ϱ as shown in Equation (9):

$$H_t = \begin{bmatrix} h_{ssj} & h_{sfj} \\ h_{sfj} & h_{ffj} \end{bmatrix} = \begin{bmatrix} h_{sj} & 0 \\ 0 & h_{fj} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{sj} & 0 \\ 0 & h_{fj} \end{bmatrix} \tag{9}$$

The Optimal Hedge Ratio: Time Varying Coefficient Models and the Kalman Filter Approach

The optimal hedge ratio may be defined as the quantities of the spot instrument and the hedging instrument that ensure that the total value of the hedged portfolio does not change. This can be more formally expressed as follows:

$$V_h = Q_s S - Q_F F \tag{10}$$
$$V_h = Q_s S - Q_F F \tag{11}$$

where,

V_h = value of the hedged portfolio,

Q_s = quantity of spot instrument,

Q_F = quantity of hedging instrument,

S = price of spot instrument,

F = price of hedging instrument,

If, $\Delta V_h = 0$

Then, $\dfrac{Q_F}{Q_S} = \dfrac{\Delta S}{\Delta F}$

$$h = \frac{Q_S}{Q_F} \text{ so that, } h = \frac{\Delta S}{\Delta F} \tag{12}$$

where, h = hedge ratio

The hedge ratio, therefore, can be represented as the coefficient in a regression of the price of the spot instrument on the price of the hedging instrument. This coefficient, however, may be more appropriately represented as time varying rather than static. It has been argued in the literature that the true relationship between economic or financial variables can be better captured by time varying parameter (TVP) models rather than fixed parameter (FP) models. One reason for this is based on the so-called Lucas (1976) critique, which states that agents rationally anticipate policy changes or the effect of unexpected events and therefore change their behavior correspondingly. Other reasons, as pointed out by Engle and Watson (1987) and Hatemi-J (2002), is that the data generating process could change due to changes in non-observable factors such as expectations. Also, as pointed out by the same authors, certain models may be misspecified partly because the non-whiteness of their error terms is due to the time varying nature of the coefficient. Finally, it has been shown that TVP models have superior forecasting properties (Phillips, 1995; Brown *et al.,* 1997).

Thus, it is important to test whether the hedge ratio is one that is captured by a FP model and therefore constant, or one that is better estimated by a TVP model and therefore time varying. Within this context, the study proposes to estimate the following two alternative models as a basis for calculating the optimal hedge ratio:

$$S_t = a + h\,F_t + e_t \qquad (13)$$

$$h_t = h_{t-1} + v_t\,,\ t = 1,\ldots\ldots\ldots,\ T \qquad (14)$$

Thus, it is important to test whether the hedge ratio is one that is captured by a FP model and therefore constant or one that is better estimated by a TVP model and therefore time varying. Within this context, the study proposes to estimate the following two alternative models as a basis for calculating the optimal hedge ratio: where T is the final time period. Equation -13, represents a fixed coefficient model where a and h are fixed coefficients to be estimated and e_t is the error term, which is assumed to be white noise with zero mean, constant variance and no autocorrelation. Hence, in Equation 13, the hedge ratio is static. On the other hand, Equation- 14 is a TVP model where the hedge ratio is time varying. The first part of Equation 14 is known as the observation (or measurement) equation and the second part as the state (or transition) equation. The state equation describes the dynamics of the coefficient h, which is assumed to follow an autoregressive process of the first degree. The error terms u and v are assumed to be independent white noise processes. This state space model can be estimated by applying the Kalman Filter. The time path of the estimated hedge ratio can then be traced.

Results and Discussion

The future and spot price of Soybean have been considered for the contracts maturing in December, 2011; June, 2012; December, 2013; April, 2013. Augmented Dickey Fuller (ADF) test was applied and the result is reported in table 1. It is seen that, in all the spot and future series, there is presence of unit root indicating nonstationarity of the series. Accordingly, all the series were differenced one time and again application of ADF test showed that the differenced series became stationary.

Table 1: Augmentd Dickey Fuller Test for stationarity of data series

Future and Spot price series	Original series p-Value	Differenced series p-Value
FT_11.7.2011 to 20.12.2011 (Dec.2011)	0.4168	<0.002
FT_ 10.1. 2012 to 20.6.2012 (Jun 2012)	0.8971	<0.004
FT_10.7.2012 to 20.12.2012 (Dec.2012)	0.7736	<0.003
FT_10.11.2012 to17.4.2013 (April-2013)	0.9863	<0.001
ST_11.7.2011 to 20.12.2011 (Dec.2011)	0.1465	<0.006
ST_10.1. 2012 to 20.6.2012 (Jun 2012)	0.9384	<0.008
ST_10.7.2012 to 20.12.2012 (Dec.2012)	0.859	<0.007
ST_10.11.2012 to17.4.2013 (April-2013)	0.946	<0.005

FT=Future price series, ST=Spot price series.

As described in previous section, the hedge ratio has been estimated for all the contracts by using OLS method i.e. constant hedge (CH), GARCH-BEKK, GARCH-VECH and Kalman filter methodology. The estimated hedge ratio by above methodology has been plotted for each contract in figure 1-4. It is evident from the figure that the hedge ratio is not constant throughout the time;

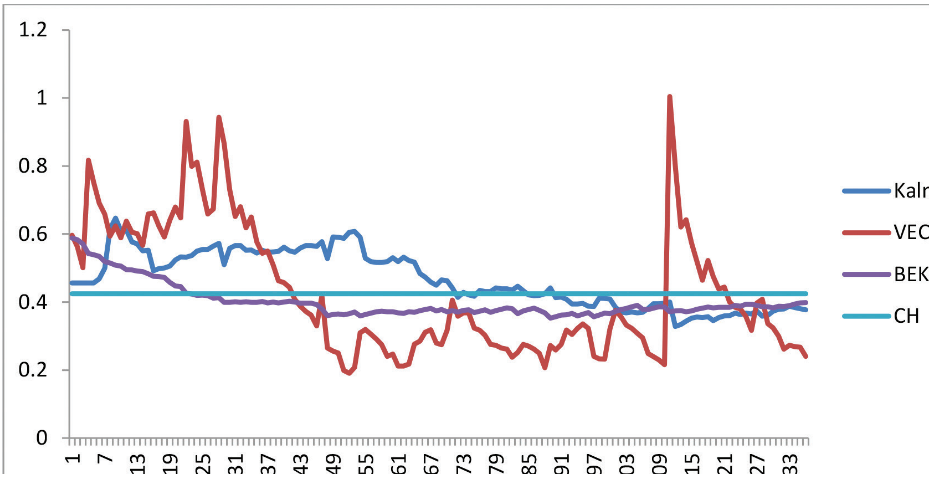

Fig. 1: Future contract maturing December-2011

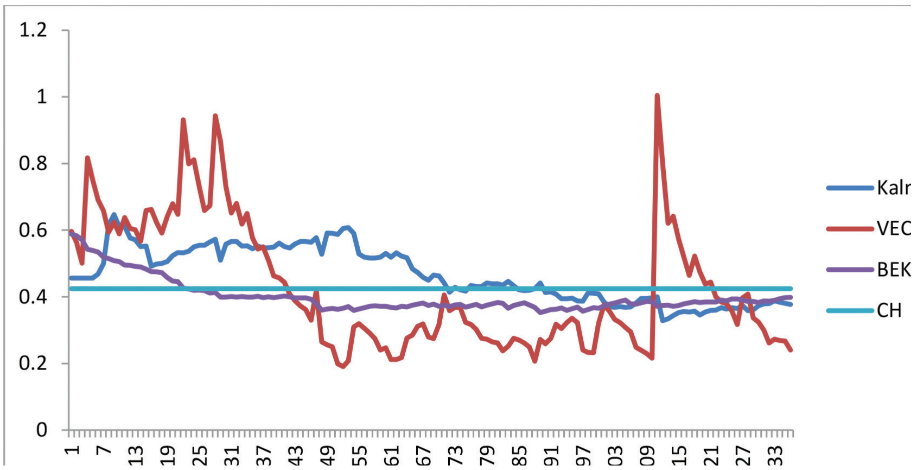

Fig. 2: Future contract maturing June-2012

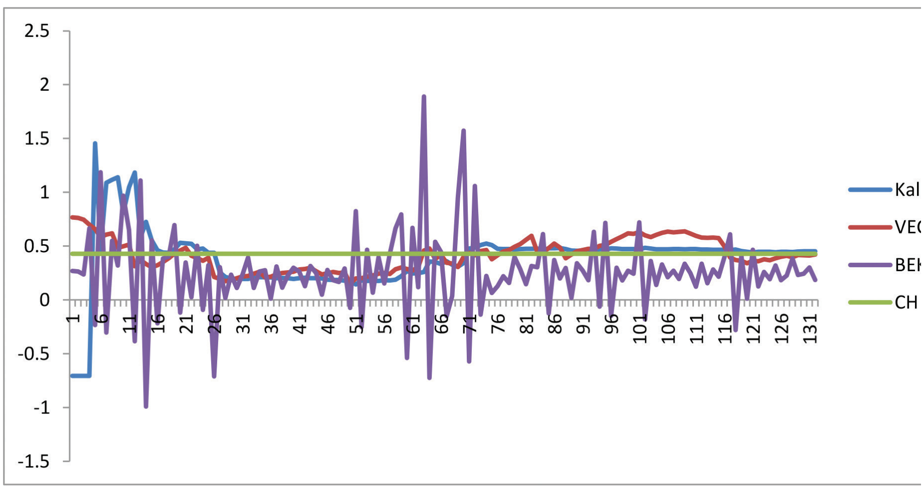

Fig. 3: Future contract maturing December-2012

Fig. 4: Future contract maturing April-2013

rather it is dynamic in nature as depicted by GARCH and Kalman filter methodology.

If this time varying nature of the hedge ratio is a guide of what is going to happen in the future, there is therefore difficulty in terms of hedging because this implies that for the hedge to work, the hedged portfolio must be rebalanced on a period-by-period basis. This may involve huge transaction costs and therefore it may not be worth using this particular instrument (futures contract) for hedging. Thus, the investor may search for a more suitable hedging instrument subjecting this instrument again to the procedure that has been demonstrated.

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